

Summarising, paraphrasing and critiquing

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1. Introduction

In order to be able to summarise and critique a topic you have to be sufficiently familiar with the topic. Since the only topic I am familiar with in sufficient depth is mathematics I will present examples relating to mathematics and statistics.

2. Summarising: Initial examples

2.1. Example 1: Statistics – On means, medians and modes in statistics

Consider the following text (which I have invented):

Here we describe the mean and median as ways of finding averages in data. The mean is a way of calculating the average value of data using simple arithmetic, whereas the median does not rely on arithmetic. Instead, the median relies on ordering the data from smallest to largest and then identifying the middle value. This middle value is a form of average because it represents the value that is most likely.

Focusing more closely on the nature of data itself, we find that there are data values called outliers. Outliers can be defined as data values which lie significantly outside the main trend of the data. A single outlier in a data set can significantly change the value of the mean compared to the mean calculated without the outlier. On the other hand, the median is not affected by outliers. When data is arranged in order the middle data values remains in the middle, irrespective of how large or small the outlier is.

Now consider the following summary and paraphrase to these two texts:

1. *Summary*: The text above mentions two ways in which we can find the average of a data set, namely the mean and the median. It also describes how outliers affect the result of the mean and median.
2. *Paraphrase*: Two standard measures of central tendencies are the mean and median. The mean is found by summing the data and then dividing by the number of data. In other words, it is the usually understood way of finding an average. On the other hand, the median relies on the data being put into ascending order, from which we locate the middle value. Because the value is in the middle of the ordered data set it is considered the most common value.

In terms of the data values themselves we find that there are such things as outliers. Outliers are values which are considered unusual because instead of following the main trend of the data they lie at an abnormal distance from all other values. They are considered extreme values since they stand out greatly from the overall pattern of the data set. Outliers can significantly affect the value of the mean but not the value of the median since the ordering of data is not affected by the size of outlying values.

Questions

By comparing the content of my summary and paraphrase above, consider the following questions:

1. What is it that makes the summary a summary? What is it that makes the paraphrase a paraphrase?
2. Is it possible to give more detail to the summary above, and it still be a summary? If so, to what extent can we include more detail?
3. Is it possible to shorten the paraphrase above, and it still be a paraphrase? If so, to what extent can we make the paraphrase less detailed?

2.2. Example 2: Mathematics (ODEs in modelling)

The following text is adapted from “Rethinking pedagogy for second-order differential equations: a simplified approach to understanding well-posed problems”, Christopher C. Tisdell, *International Journal of Mathematical Education in Science and Technology*, 2017, 48:5, 794-801.

“An important question arising from the modelling and analysis of differential equations involves asking whether or not the given differential equation subject to some initial conditions is well posed. That is, we want to know if the problem has exactly one solution and to have simple sufficient conditions at hand which we can apply to a wide range of cases to verify the ‘well posedness’ of a given problem. If the problem has no solution, or multiple solutions, then it is not well posed from a modelling point of view and it must be discarded and a new model formulated.”

One way to analyse the text in order to write at the level of a summary is to identify which parts present details of the topic, and which parts present the topic more generally. Below I have highlighted in light grey those parts which present more detail.

An important question arising from the modelling and analysis of differential equations involves asking whether or not the given differential equation subject to some initial conditions is well posed. That is, we want to know if the problem has exactly one solution and to have simple sufficient conditions at hand which we can apply to a wide range of cases to verify the 'well posedness' of a given problem. If the problem has no solution, or multiple solutions, then it is not well posed from a modelling point of view and it must be discarded and a new model formulated.

Then, if I can ignore /delete those aspect in light grey and still have a text which makes sense, then my text is more general. The following is the resulting text with the grey phrasing deleted.

An important question arising from differential equations involves asking whether or not the given differential equation is well posed. That is, we want to know if the problem has exactly one solution to verify the 'well posedness' of a given problem. If the problem has no solution, then it is not well posed from a modelling point of view.

The question then is, Is there a common idea to which this detail relates? Yes. That of well-posedness. So this will be the main focus of my summary, and I can write a draft summary to be

The author discusses the well-posedness of differential equations when modelling problems.

If this is a sufficiently detailed summary then I can stop there. If not, I can then return to the original text either to add more detail or to generalise given detail. I then continue this process of adding more detail, or generalising, in stages all the while making sure my text remains a summary, and not turning into a paraphrase. My second attempt at a summary could therefore give:

When using differential equations to model problems the author discusses the well-posedness of these equations in respect of type of solutions these equations give.

Now note that the phrasing "the type of solutions these equations give" is a generalisation of the phrasing "has exactly one solution and to have simple sufficient conditions at hand" and

“the problem has no solution, or multiple solutions,” I have to write such a generalisation in order to write in a summary style. As for paraphrasing, I might write

The well-posedness of second- or higher order differential equations (DEs) has usually been addressed by reducing the DEs to a system of first order DEs. This is done in order to determine whether or not the original DE has exactly one solution. From a modelling perspective, should the DE not have a unique solution then a new model needs to be created.

From the perspective of teaching and learning about well-posedness this reduction of second- or higher-order DEs to a system of first order DEs makes for more work than necessary. A more direct approach to answering the question of well-posedness is presented here.

Questions

By comparing the content of my summary and paraphrase above, consider the following questions:

1. What is it that makes the summary a summary? What is it that makes the paraphrase a paraphrase?
2. Is it possible to give more detail to the summary above, and it still be a summary? If so, to what extent can we include more detail?
3. Is it possible to shorten the paraphrase above, and it still be a paraphrase? If so, to what extent can we make the paraphrase less detailed?

3. Commentary about summarising and paraphrasing

We can summarise texts we ourselves have written (such as in the summary of our own essay) or texts written by someone else. Ditto for paraphrasing. How do we make sure our writing is *summary writing* or *paraphrase writing*? By

- *its scope*: Summaries are generalistic in content. This means that they address only the main themes or ideas of a text, and omit most details. As such, summaries are much more general descriptions of the original text. For example, summaries do not contain quotes from the original paper, nor the authors' opinions, nor certain levels of detail of the topic; Paraphrases are much more focused in content than summaries. This means that one tends to write at the same level of detail and depth (more or less) as the original text.
- *its language*: Summaries and paraphrases both use reference-type language (not to be confused with references as a list of books and papers included at the end of an essay). But summaries are more generalistic in style of writing whilst paraphrases are detailed. This means that we use alternative phrasing, and we construct sentences or paragraphs differently to that of the original text, whilst keeping the same emphasis and meaning of the original text.

The following are examples of phrasing used as reference-type language one can use when summarising:

- 1a) "Prior work by ... reveals that ...": this references other peoples' work. After this we summarise the main ideas of the paper;
- 2a) "Earlier in this paper I stated that ...": this references something I said earlier in the paper. After this I summarise my main ideas;
- 3a) "It is generally accepted that ..."; this references an idea, theory, opinion, etc., that the community of experts agree upon. After this we summarise the generally accepted thing;
- 4a) "In summary we have ...": this references something previously stated/written, either in the same paper or from another paper; After this we actually write the summary we say we are going to write;
- 5a) "Author X highlights similarities and difference between ...": this references a specific person's critique; After this we summarise the similarities and differences;

6a) “However, as discussed in Smith (2000) ...”: this references the comments made by a specific author. After this we summarise the author’s main idea.

etc. Sentences involving the type of phrasing above can then be seen to be generalistic in style, i.e. they do not mention any particular focus. Examples of such focus are shown below:

1b) “Prior work by *Smith (2000)* on the mean of random samples taken from a single population reveals that *these means have a natural variation*”. Two possible aspects of detail missing here could relate to

- i) the distribution of the population: is it normally distributed? Is it skewed? Etc.;
- ii) the size of the samples taken: too small a sample size adversely affects the sample means. There is a greater variety in the means for small sample sizes compared to large sample sizes;

Here 1b) is classed as a summary because of the generality of the phrasing used and the focus on the main ideas without going into specific about these ideas.

3b) “It is generally accepted that *the mean is susceptible to outliers whereas the median is not*”. Two possible aspects of detail missing here could relate to

- i) how extreme the outliers are: the more extreme the outlier, the greater its effect on the mean;
- ii) how the median is unaffected by extreme values: since data is arranged in ascending order the size of the outliers has no effect on the middle value of that ordered data.

Here 3b) is classed as a summary because of the generality of the phrasing and the focus on the main ideas without going into specific about these ideas.

The table below illustrates more examples of types of phrasing and sentence development. The aim of this table is to show you examples on an *underlying principle* of what constitutes *summary* language and description. This underlying principle is what you should aim to learn and understand. Then you will know *how* to paraphrase or *how* to write a summary, and you will only need to learn individual vocabulary, terminology, and phrasing in order to paraphrase or summaries in *your own words*.

<p>{ prior previous past ... }</p>	<p>{ studies work research data ... }</p>	<p>by author A on ...</p>	<p>{ reveals indicates shows illustrates ... }</p>	<p>{ important fundamental crucial significant ... }</p>	<p>{ similarites differences changes deviatiions ... }</p>	<p>in ... with respect to ...</p>
<p>Author A</p>	<p>{ describes highlights criticises defines ... }</p>	<p>{ similarites differences changes deviations ... }</p>	<p>in ... with respect to ...</p>	<p>{ as a result of due to because of by reason of in order to account for ... }</p>	<p>...</p>	

Two examples of suitable language for writing summaries

When one is paraphrasing one generally tends to write at the same level of detail and depth (more or less) as the original text, but this time using one's own words. This means that we use alternative phrasing, and we construct sentences or paragraphs differently to that of the original text, whilst keeping the same emphasis and meaning of the original text.

For example, consider the following text as a primary piece of writing:

In his paper we discuss the mean as a measure of central tendency, explaining that the advantage of this is that, because all the data values are used in finding the mean, taking the mean for different samples of a population tends to give similar results. This indicates that the mean is robust, namely that it resists very well any fluctuations between different samples.

we can paraphrase this as illustrated below whilst retaining the essential meaning of the text:

- The advantage of using the mean as a measure of central tendency are discussed. Examples of finding means for different samples from the same population are presented, the results of which indicate that these means are similar. Such an outcome illustrates that there is little fluctuation between different samples. Because of this the mean is said to be robust.

or

- This paper describes, with examples, how it is that means calculated from different sample of the same population is robust. Robustness can be defined as the ability of data to resist fluctuation to a certain degree. This proves to be one of the great advantages of this measure of central tendency.

Compare the above paraphrases with the following summaries:

- The particular type of average known as *mean* is discussed with reference to the advantages it brings when used in analysis data.

or

- A particular quality of the mean data, known as *robustness*, is presented along with its benefits.

Exercises

- 1) Do the summaries above keep the essential meaning of the original text?
- 2) Do the summaries above focus on the same theme as each other? As the original text?

4. So what is a summary and a paraphrase?

So what is the difference between a summary and a paraphrase? Well,

1. The aim of a summary is to report in a brief and yet accurate manner the main idea, objective, methodology, results, and success/failure of intended aim of the original paper. The goal of summary is not to offer an evaluation or opinion of the original article. A summary is far more concise than the original paper. It is a self-contained piece of writing which is fully formed and able to make sense on its own.

As such, the summary of example 1 on p1 is short and mentions only the main or essential theme of the text.

2. The aim of a paraphrase is to demonstrate your own understanding of a text, and do so in your own words. Paraphrases are longer than summaries and approximately the same length and level of detail as the original text. This requires two things: i) an understanding of the topic of the text, ii) a certain ability at using language to write one's understanding.

As such, the paraphrase of example 1 on p1 goes into more detail than the summary and does so in my own words rather than repeating phrases or sentences of the text. My paraphrases uses synonyms but is not simply just this. Nor, generally, does my paraphrase reorder phrases or sentences, or copy phrasal or sentence structure of the original text. I do this latter only if I am quoting *or if the phrasing is standard technical phrasing used in the discipline* (as in "significance testing", "confidence intervals", "performing a *t*-test at the 5% confidence level", etc.)

It is only by doing a lot of reading that you come to know what paraphrases and summaries look like, how they are written, and how to write them yourself. This leads us to the fact that a summary or a paraphrase is seen to be a summary or a paraphrase by the use of a particular type of vocabulary constructed via phrasing and sentences and paragraphs in a particular way.

5. Critiquing

We now move onto the aspect of critiquing a text. Generally, one cannot critique a text without first summarising it. Hence, in order to see critiques in context, summaries will be showed alongside in the examples below. We start with an example before moving on to discussing the nature of a critique. This will give some general concrete experience when it comes to developing a more thorough understanding of what a critique is.

5.1. Example 1: Statistics – On means, medians and modes in statistics

Consider the following text which I have invented myself:

When it comes to a basic analysis of data Smith (1980) uses the mean as his measure of central tendency. He explains that the advantage of this is that, because all his data values are used in finding the mean, taking the mean for different samples of a population tends to give similar results. This indicates that the mean is robust, i.e. it resists very well any fluctuations between different samples.

On the other hand, Jones (1990) makes use of the median as his measure of central tendency. This is because does not rely on arithmetic but on ordering the data from smallest to largest and then identifying the middle value. This middle value is a form of average because it represents the most common value among the data.

We can then write the following summary and critique:

- *Summary:* This text summarises and critiques two different authors' choice of measures of central tendency. On author, Smith (1980), uses the mean and explains the advantages and disadvantages using this measure, whilst the other author, Jones (1990), uses the median and also explains the advantages and disadvantages using this measure.
- *Critique:* Whilst the description above is correct in terms of describing the mean and the median, it omits certain crucial aspects of their use, aspects of which can significantly influence the outcome of any statistical test. Specifically, using the mean as an average can be problematic since the mean is sensitive to outliers. The further away the outlier (i.e. the larger the value of the outlier) the more it will affect the mean, resulting in a value of the mean which is not representative of the "middle" of majority of the data. So, one single value can significantly skew the value of the mean away from the most representative average. However, this is not considered in the text above.

The problem of outliers can be remedied by using the median, as Jones (1990) does. but this too has a problem, namely that, because the median is not calculated arithmetically (and therefore does not use each data value), it is easily affected by the type of sample we take from the population.

Furthermore, when performing basic analysis on data one never simply uses a measure of central tendency (the mean or the median). One also measures the spread

of the data. This is would either be measuring the standard deviation of the data if one is using the mean as an average, or using the inter-quartile range if one is using the median as an average. Neither author mentions using any measure of spread, which is a serious omission when analysing data for the information it can provide.

Exercise:

Is my summary a summary? Is my critique a critique?

- a) If not, why not? What is wrong with my summary and/or critique? How would you correct these so that they read as a summary and critique?
- b) If so, what is it that makes it a summary and critique? What type of language or phrasing or intention am I using in both of these forms of writing?

5.2. Example 2: Mathematics (ODEs in modelling)

Consider the following text which is an extension of example 2 on p2:

“An important question arising from the modelling and analysis of differential equations involves asking whether or not the given differential equation subject to some initial conditions is well posed. That is, we want to know if the problem has exactly one solution and to have simple sufficient conditions at hand which we can apply to a wide range of cases to verify the ‘well posedness’ of a given problem. If the problem has no solution, or multiple solutions, then it is not well posed from a modelling point of view and it must be discarded and a new model formulated.

For over 70 years, learning and teaching approaches concerning the well posedness of second-order (and higher-order) initial value problems (IVPs) have involved a significant detour. Scholars have reduced second-order (and higher-order) problems to first-order systems of equations through a transformation and then performed an analysis on the resultant system. We show that this excursion is unnecessary and present a direct approach regarding second- and higher order problems.”

The second paragraph of this text represents the critique. The key phrasings, and development of the paragraph, which suggests this is highlighted in the text below:

The first sentence

“For over 70 years, learning and teaching approaches concerning the well posedness of second-order (and higher-order) initial value problems (IVPs) have involved a significant detour ...”

This can be more generally seen to represent

doing something (learning and teaching)
on/about something (well-posedness)
has the impact, or effect of (significant detour).

The text then goes on to describe what this “detour” is, namely that

“Scholars have reduced second-order (and higher-order) problems to first-order systems of equations through a transformation and then performed an analysis on the resultant system.”

This is not a critique, simply a description. In other words,

Scholars have done something (reduced problems)
in a certain way (by a transformation)
and then *taken action as a result* (performed an analysis).

The authors then suggest an alternative way which they consider better:

“We show that this excursion is unnecessary and present a direct approach regarding second- and higher order problems.”

This sentence (the last of the paragraph) is a critique of the previous sentence, identified by the language “We show that this excursion is unnecessary”. The author then goes onto say that he has an alternative way of doing things.

Also, as another example consider the first paragraph with the last sentence omitted. As such no reference is made to models which are not well posed. Someone reading the first paragraph could then write the following critique:

The author talks about the well posedness of problems modelled by ODE. However, the author has not taken into account the fact that that not all problems

modelled by ODEs have a unique solution. The way in which the problem is modelled may give rise to no solution or to multiple solutions. Addressing this issue can then provide for a more contextual understanding of the nature of solutions to ODEs, and thus to the nature of modelling by ODEs.

5.3. Example 3: Statistics – On correlation coefficients

The following text is taken from “The Absolute Correlation Coefficient”, Christopher Bradley, *The Mathematical Gazette*, Vol. 69, No. 447 (Mar., 1985), pp. 12-17.

The two most common measures of central tendency and dispersion statistics are the mean and standard deviation on the one hand, and the median and absolute deviation on the other. For most purposes the former measure is preferred for two very good reasons; the first is that the squares of quantities are easier to handle analytically than their moduli; and secondly for all the common symmetrical distributions [...] if a sample is taken to estimate the central value, then the mean of that sample has a smaller variance than the median, and is therefore relatively more efficient as an estimator of the central value of the parent population.

The preference for the mean and standard deviation is so pronounced by the time the topic of correlation is studied that little, if anything, is ever written about a possible analogue of the product moment correlation coefficient. So [...] I thought it might be interesting to show that a theory of absolute correlation can be constructed, which is based on the measure of median and absolute deviation.”

We can then write the following summary, paraphrase and critique:

- *Summary:* This text describes two reasons why the mean and standard deviation are preferred over the median and absolute deviation as measures of central tendency and dispersion. Then, focusing on correlation the author wishes to develop a theory of correlation coefficient based on the median as opposed to the usual one based on the mean.
- *Paraphrase:* The mean and standard deviation dominate current statistical practice for two reasons: i) the squaring effect makes mathematical analysis easier, and ii) the sample mean will be a better estimate of the population mean and the sample median. By the time we get to studying correlation coefficients the median and absolute

deviation have been completely sidelined as measures of centre and dispersion, so that we are left with only the mean and standard deviation in any future statistical theory. the author wishes to correct this by presenting a version of correlation coefficient which corresponds to the median and absolute deviation.

- *Critique:* Absolute deviation uses, by definition, the absolute value function. The great problem with such a formulation lies in the fact that the absolute value function is not differentiable at $x = 0$. Any theory which relies on the derivative of the absolute deviation will therefore need to exclude that point at which the function is not differentiable. This means that it will be impossible to analyse any correlation between two data sets at that specific point.

Exercise:

Is my summary a summary? Is my paraphrase a paraphrase? Is my critique a critique?

- a) If not, why not? What is wrong with my summary, paraphrase and/or critique? How would you correct these so that they read as a summary, paraphrase and critique?
- b) If so, what is it that makes them summaries, paraphrases and critiques? What type of language or phrasing or intention am I using in my critique? Which of a. to d. have I used? If I haven't used any of these forms of analysis what form of analysis have I used?

6. The language/discourse of a critique

The examples above on presenting critical thinking (as critiques of the topics) involved certain types of vocabulary, phrasing and sentence building. The way in which this vocabulary and phrasing can be built is illustrated in the table a few pages down. The aim of this table is to show you examples on an *underlying principle* of what constitutes critique language and description. This underlying principle is what you should aim to learn and understand. Then you will know *how* to write in a critical manner, and you will only need to learn individual vocabulary, terminology, and phrasing in order to express *your own* criticality.

For example, given the following critique text,

- Effective teaching does not mean providing only the explanations that make sense for experts, but also introducing conceptualizations that make sense from the point of view of learners.

we can rewrite this as illustrated below whilst retaining the features and essence of a critique:

- Beyond using teaching simply to impart the standard meaning of logarithm, it is important to support learners in developing their own understanding of the concept of logarithms,

or

- An important element in learning about logarithms is the way in which learners make sense of this in their own way. This is something which is not considered in the standard way of teaching logarithms whereby only the expert definition is given.

Similarly, given the following,

- However, research on primary and secondary students has already documented difficulties in reasoning about quantitative data when it is provided in the aggregate, as in histograms, line plots or other frequency graphs (Friel & Bright, 1995; McClain, 1999; Watson, et al., 2003). Taken together, the existing body of research indicates that students entering college may have only a superficial understanding of center and variability, and are likely to have particular difficulty extracting information about those features when data are presented in graphical form.

We can rewrite this as illustrated below whilst retaining the features and essence of a critique

- Previous assumptions about college students' understanding of measures of center and variability have proved incorrect. It has been shown that such students seem to have little understanding of these statistics. As a result, they have difficulty identifying such measures when data are presented in graphical form. Specific research by Friel & Bright, 1995; McClain, 1999; and Watson, et al., 2003 has found that primary and secondary students have difficulties in reasoning about quantitative data when such data is presented in aggregate form, as in histograms, line plots or other frequency graphs.

or

- The ability to understand measures of center and variability can be seen in the way students recognise, or not, such features in data presented in aggregate form, as in histograms, line plots or other frequency graphs. However, the assumption that students enter college with an adequate understanding of these features has been shown not to be the case. For example, research by Friel & Bright, 1995; McClain, 1999;

and Watson, et al., 2003 has found that primary and secondary students have difficulties in reasoning about measures of center and variability.

Below is an artificial critique using key vocabulary and phrasing in order to illustrate the critique-ness of the critique below. Bear in mind that the example below is extreme, but it is designed to highlight the nature of critique-ness:

This method is sufficiently inaccurate as to be flawed and unsuccessful because of its limited use in the relatively narrow domain, and its irrelevant use of an unscientific model.

{ person method process ... }	X is a	{ important fundamental crucial decisive ... }	{ aspect element issue ... }	of/in/about	...	because ... due to ... by reason of ... in that ...
	The	{1, 2, 3...}	{ important fundamental crucial decisive ... }	{ aspects elements issues ... }	of/in/about ... are	because ... due to ... by reason of ... in that ...

Table: Examples of critiques type sentences using certain types of vocabulary and phrasing.

Other examples of language which can be used when writing in a critique manner is shown below

might have / could have / would have	In other words the analysis could be ...	An alternative approach might/would be to ...	Not only has/have ... but also ...
Particularly important /relevant/useful was/were ...	Of less significance was/were	It may be that ...	this could be explained by ...
certain changes/ additions/elaboration might be needed ...	An explanation/description /example of ... would be appropriate	The author then does ... but without doing ...	However, if this approach were used/adopted ... then ...

7. Summary of linguistic structures that can be used in critiques

The following illustrate the type of linguistic features one can use when critiquing. It is taken from C1/C2 IPM content you may have already seen.

(1) **3rd conditional or past unreal conditionals**, e.g.:

- “The analysis *might have* been stronger if ...”
- “The writer *could have* focused more on ...”
- “The study *would have* achieved greater accuracy if ...”

(Note - In a critique the *if* clause is often placed second in the sentence, after the main clause¹. Why do you think this is?)

(2) **Inversions** when a negative or an adjectival phrase begins a sentence, e.g.:

- “*Not only has* this study challenged previous findings, it has also...”
- “*In no part* of the methods section *do* the authors specify precisely what ...”
- “*Particularly salient were* the observations on ...”
- “*Of less significance were* the findings ...”

(Note – Inversions foreground or give special emphasis to the information/idea located at the beginning of the sentence. Why might a writer choose to do this?)

(3) **Hedging/Boosters** to make clear precisely how weak/strong a claim is, e.g.:

- “This *arguably* goes further than ...”
- “It *may be* that this factor ...”
- “... and it *could be* explained on the basis that ...”
- “and this is *certainly* a major advance ...”
- “... the authors have *clearly* established ...”

(4) **Attitude markers** revealing the attitude of the writer of the critique to its subject-matter, e.g.:

- “*Surprisingly* the author did not consider ...”
- “It is *difficult to understand why* ...”
- “... is *particularly interesting*.”

¹ Swales, J. and Feak, C. (2012). *Academic Writing for Graduate Students*. 3rd ed., Ann Arbor, MI: University of Michigan at 260

(5) **Self-mentions, e.g.:**

- "... but, *as it seems to me*, this ..."
- "*I was not persuaded* by this argument."
- "*I believe* ..."
- "Nevertheless, *I would argue* that this approach ..."

(Note - The use of self-mentions varies considerably from discipline to discipline and likewise opinions about the stylistic appropriacy of self-mentions can vary (sometimes considerably) from tutor to tutor within a particular faculty or department. Therefore you should check with your tutor or department whether it is considered acceptable to use self-mentions when writing a critique *before* you start to write.)

(6) **Choice of lexis**

The table on the next page contains a list of vocabulary items which are commonly used when writing a critique².

Verbs	Adverbs	Adjectives	Nouns
account for aid analyse answer appear assert collect combine complete describe employ exhibit fail predict raise represent review seem succeed suffer from suggest wonder	accurately completely correctly currently enough exactly fully inaccurately incorrectly insufficiently later necessarily really relatively successfully sufficiently unfairly unsuccessfully	accurate ambitious apparent beneficial careful competent complete complex correct detailed difficult effective extra fair flawed good important impressive inaccurate incorrect ineffective innovative insignificant insufficient interesting likely limited little modest obvious potential preliminary reasonable reliable remarkable restricted scientific serious	accuracy analysis aspect assumption collection consideration difference difficulty effect element factor flaw growth impact implication importance inaccuracy increase information insight model reduction significance source site tool

² Adapted from Swales & Feak (2012) (op. cit.); Nesi & Gardner (2012) (op. cit.)

Verbs	Adverbs	Adjectives	Nouns
		significant similar simple small successful sufficient suitable unfair unimportant unlikely unreasonable unreliable unsatisfactory unscientific unsuccessful unusual useful	

Exercise

How many of the linguistic features above have been used in the critiques of examples 1, 2, and 3 above?

8. Some criteria for developing a critique: What can be critiqued

8.1. What one can critique

Anything can be critiqued:

- The literature;
- Someone’s interpretation of the literature;
- The design of the experiment, the apparatus used, the experimental procedure;
- The data collection methods;
- The analysis of the data;
- The theoretical framework/model/methodology used;
- Any assumptions made;

For example,

- **Critiquing the methodology used:** How has the author collected the data? How have they analysed the data? What methods are they using to analyse the data?

Examples

1) Descriptive statistics can only be used to categorise and summarise the data collected. It cannot be used to infer or generalise anything about the larger set of data from which your data comes from. However, descriptive statistics does allows

us to see an overall pattern or trend (such as by the use of graphs).

- 2) Inferential statistics allows us to make generalisations and inference. In this way we can take a sample, study it and reasonably extrapolate or infer the behaviour of the population from which the sample came. There is, however, a disadvantage to inferential statistics in that the analyses and results are never accurate. There will always be errors in the final results, and the conclusions based on these results will only be approximate.

- **Critiquing the literature review**

8.2. How one can critique

- **Comparing and contrasting:** Look for similarities and differences between what different authors have said or done.

Examples

- 1) Between the logarithm function and the exponential function, the most common approach is to define the logarithm first (as an integral) and then define the exponential as the inverse of the logarithm. However, such an approach requires the student to first learn calculus before they can use these two functions, thus leaving the use of these function quite late in the student's learning.

On the other hand the exponential function can be introduced after the binomial theorem, which is part of algebra, and comes much earlier in the student's learning.

- 2) Although both the mean and the median are both aimed at finding the average value of the data, a major difference can be observed between the two measures, namely that the former is sensitive to outliers whereas the latter is not."

There are two points to note here:

- i) firstly, note the language of “similarity and difference”.
- ii) secondly, look for synonyms and antonyms of such language (see the sources you have been told about in other classes, as well as a thesaurus, as well as online phrasebanks)

- **Identifying flaws or weaknesses and strengths or positive:** Look for problems, limitations, assumptions, etc in theories, arguments, methods, practice, etc.

Examples

- 1) “In his use of the mean Smith (1990) has failed to take into account the aspect of outliers in his data. As a result of this his calculation for the mean is not as representative of the middle as it could be. One way of overcoming this flaw could be to ...”
- 2) “However, Jones (1995) does not assess the effect of ignoring the outliers in his data when calculating the line of best fit.”
- 3) “The use of the z-test on this data has the limitation that it can only be applied to data which is (approximately) normally distributed.”

- **Offer constructive suggestions:** Offer suggestion for correction, improvements, etc.

Examples

- 1) “A much more accurate trend curve could have been developed if the author had used an exponential fit instead of the linear fit to his data.”
- 2) “It would have been useful for the author to calculate two lines of best fit: one containing the outlying data, and one ignoring the outlying data. In this way a comparison could have been made between the two best-fit lines in order to ...”
- 3) “Given that the data used is skewed, a more appropriate measure of central tendency would have been the median”.

- Similarities and differences,
- advantages and disadvantages, etc.

Shift these two away from evaluation? So, what is it that goes into eval, and what goes into critique? See item 2. P25!!

These are only four aspects of what can be considered for a critique. By reading widely you will come to see other aspect which can be considered.

9. Critiques from the literature

9.1. Example 1: Statistics in medicine

The following text is from “A critique of statistical hypothesis testing in clinical research”, S. Raha, *Journal of Ayurveda & Integrative Medicine*, July-September 2011, Vol 2, Issue 3. The critique vocabulary or phrasing is highlighted underlined. Also, certain parts are annotated wherefrom you can refer to the relevant comment.

Many have documented the difficulty of using the current paradigm of Randomized Controlled Trials (RCTs) to test and validate the effectiveness of alternative medical systems such as Ayurveda. [...] ... the two main worldviews of probability are that of the Bayesian and the frequentist. The frequentist worldview is a special case of the Bayesian worldview requiring the unrealistic assumptions of knowing nothing about the universe and believing that all observations are unrelated to each other¹. Many have claimed that the first belief is necessary for science, and this claim is debunked by comparing variations in learning with different prior beliefs². Moving beyond the Bayesian and frequentist worldviews, the notion of hypothesis testing itself is challenged on the grounds that a hypothesis is an unclear distinction, and assigning a probability on an unclear distinction is an exercise that does not lead to clarity of action.³”

Notes

1. The language “unrealistic assumptions” is critique language, specifically an attitude marker, and the whole sentence describes what the unrealistic assumptions are.
2. The language “is debunked” is critique language, specifically an attitude marker, and the whole sentence explains how “the first belief is necessary for science” will be debunked.
3. Within the context of the sentence the language “is challenged” is critique language, with the rest of the sentence explain the reasons for the challenge, again using critique language via “unclear distinction” and “does not lead to”.

9.2. Example 2: Scientific evidence

The following text is from “Scientific Evidence: Creating and Evaluating Experimental Instruments and Research Techniques”, William Bechtel, *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association*, 1990, Vol. 1990, Volume One: Contributed Papers (1990), pp. 559-572. The critique vocabulary or phrasing is highlighted underlined. Also, certain parts are annotated wherefrom you can refer to the relevant comment.

“The question of how scientific hypotheses and theories should be evaluated in light of evidence has been a central question in philosophy of science. Far less attention has been given to the questions of how evidence is developed and is itself evaluated¹. From this neglect, one might assume that the processes by which scientists develop and evaluate evidence are unproblematic.”²

Notes

1. The language “Far less attention has been given” is critique language, specifically an inversion (the author could have written “The questions of how evidence is developed and is itself evaluated has received far less attention”). The whole sentence specifies those aspects needing more attention (with the assumption that the paper will address this omission?).
2. The language “From this neglect” is critique language, specifically an attitude marker, and the rest of the sentence describes a possible assumption which may result from such a lack of attention.

Similarly, “one might assume” is critique language, specifically hedging, and is trying to highlight or clarify an aspect of the process scientists use when it developing and evaluating evidence.

9.3. Example 3: Design automation

The following text is from “Is Design Automation a Feasible Tool for Improving Efficiency in Production Planning and Manufacturing Processes?”, Jim Lindholma, Kerstin Johansen, 8th Swedish Production Symposium, SPS 2018, 16-18 May 2018, Stockholm, Sweden, *Procedia Manufacturing* 25 (2018) 194–201. The critique vocabulary or phrasing is highlighted underlined. Also, certain parts are annotated wherefrom you can refer to the relevant comment.

“The planning of production and manufacturing modules in an industry is referred to as production planning [6], and requires consideration of many factors, including current production flow, material use, machine scheduling, order history,

and sale trends [7]. Production planning is often assisted by a packaged software, known as enterprise resource planning systems (ERP) [8], but the engineers responsible for the planning are still personally burdened to find optimal scheduling solutions.¹ Large number of product variants, and highly varied demands for these, makes the engineering task even more difficult.²

Notes

1. Within the context of the sentence the language “but” and “personally burdened” is critique language, specifically an attitude marker, where these two words are used in combination. The rest of the sentence then goes on to specify the particular burden.
2. Within the context of the sentence the language “even more difficult” is critique language, and refers to the aspects previously mentioned in the sentence. Inversion language would make the sentence look like “Even more difficult to the engineering tasks are the large number of product variants, and highly varied demands for these scheduling solutions.”

9.4. Example 4: Chemistry

The following text is from “Review of Chemical Vapor Deposition of Graphene and Related Applications”, Yi Zhang, Luyao Zhang, Chongwu Zhou, *Accounts of Chemical Research*, Vol. 46, No. 10, 2013, p2329–2339. The critique vocabulary or phrasing is highlighted underlined. Also, certain parts are annotated wherefrom you can refer to the relevant comment.

“While graphene can be prepared using mechanical exfoliation, epitaxial growth on SiC, and chemical exfoliation, chemical vapor deposition (CVD) has emerged as an important method for the preparation and production of graphene since it was first reported in 2008 and 2009. However, graphene CVD has not been adequately covered in reviews.¹ As an example, the review by Mattevi et al. was dedicated to CVD of graphene on copper and did not include discussion of CVD graphene applications.²”

Notes

1. The language “However” in combination with and “not been adequately covered” is critique language.
2. Within the context of the sentence, and the fact that this sentence continues from the previous sentence, the language “did not include” is critique language, and the rest of the sentence mentions what was not included.

9.5. Example 5: Plasma physics

The following text is from “Particle simulation of plasmas: review and Advances”, J. P. Verboncoeur, *Plasma Phys. Control. Fusion*, 47 (2005) A231–A260. The critique vocabulary or phrasing is highlighted underlined.

“The particle-in-cell (PIC) method allows the statistical representation of general distribution functions in phase space. [...] In general, the PIC method employs the fundamental equations without much approximation, allowing it to retain most of the physics.

Despite having many advantages, the PIC model also has a number of weaknesses. Perhaps the most quickly encountered of these is computational efficiency. This is a consequence of the statistical model in which numerical fluctuations converge as $N^{-1/2}$ for N particles; the PIC scheme and some modifications can reduce the constant but not the scaling. An associated problem is the difficulty in resolving the tail of the distribution, which is often poorly populated by statistical methods. It is also challenging to model large ranges of timescales, as short timescales require small time steps while long timescales require running many time steps. Similarly, large ranges of space scales present similar difficulties for the mesh size. Finally, the PIC method requires significant memory and processor resources, and for the foreseeable future this will remain the case.”

Exercise: No notes have been provided for this example. So, as an exercise, explain why the phrases underlined represent critique phrasing.

10. Summarising, paraphrasing and critiquing: A comparison

10.1. What are summaries, paraphrases and critiques?

We may describe the nature of summaries and critiques as follows (where I have deliberately repeated the description of summary of **pError! Bookmark not defined.** in order to make this more accessible when comparing with a critique):

1. The aim of a summary is to report in a brief and yet accurate manner the main idea, objective, methodology, results, and success/failure of intended aim of the original paper. The goal of summary is not to offer an evaluation or opinion of the original article. A summary is far more concise than the original paper. It is a self-contained piece of writing which is fully formed and able to make sense on its own.

As such, the summary of example 1 on p1 is short and mentions only the main or essential theme of the text.

2. The aim of a paraphrase is to demonstrate your own understanding of a text, and do so in your own words. Paraphrases are longer than summaries and approximately the same length and level of detail as the original text. This requires two things: i) an understanding of the topic of the text, ii) a certain ability at using language to write one's understanding.
3. A critique is very different to a summary or a paraphrase. In a critique we discern, or judge, from an academic point of view, some of the following aspects:
 - a. similarities and differences between two or more sources;
 - b. significant and/or insignificant aspects of the work done;
 - c. strengths and weakness, pros and cons, disadvantages or advantages of the work done;
 - d. aspects of the work which are important or not, and why they are important or not;
 - e. the usefulness of any solutions proposed;

The above categories a. to e. form a type of comparative analysis of the text. In order to carry out such a comparative analysis we must first summarise the text. So, in general a critique can't be done without first doing a summary. As a result of performing one, or more, of a. to e. above critiques can be quite long.

So, when writing a summary ask yourselves, Is the content of what I am writing actually a summary? Am I writing in a summary style? Is this the language/discourse of a summary? Ditto for a paraphrase and for a critique: Is the content of what I am writing actually a paraphrase or a critique? Am I writing in a paraphrasing or critiquing manner? Is this the language/discourse of a paraphrase or a critique?

10.2. Comparing critical thinking with a critique

We can say that

- i) critical thinking is about developing a rationale or reasoned argument about something (i.e. the existing literature, a theory, any experimental work conducted, any conclusions reached). As such critical thinking requires reflection. In this way critical thinking explains the *how and/or why* of something (such as how/why something was

done), not just the *what* of something (such as what was done, this being only a description);

- ii) a critique, at its simplest level, is an opinion about the author's work. But it is not just any opinion. It is a form of opinion which requires critical thinking about a) similarities and differences, b) pros and cons, c) limitations, flaws and benefits, d) significance, e) relevances, f) assumptions, etc. of the author's work.

Some appropriate questions which refer to critiquing include (but are not limited to)

- asking about links between ideas,
- asking about the importance and relevance of arguments and ideas,
- asking whether or not things could have been improved, changed, done differently, etc...,
- asking whether or not any limitations or assumptions or inferences were made,
- asking about any consequences, side effects or unseen reactions as a result of the work done by the author,
- asking about errors, inconsistencies, conflicting evidence, gaps, etc. in the research,
- asking about the strengths and/or weaknesses in the research,

Note that you do not need to memorise these questions, nor address all of them in a critique. These questions are simply to illustrate the type or nature of questions that we can ask when we are critiquing. Usually the text/paper itself will "tell" you what it is you might want to critique.

So we might say that

Critique = critical thinking

+ critique criteria (items a) – f) of ii) above)

+ critique language.

A summary of the difference between summaries (which are a form of descriptive writing) and critiques is shown in the table below (adapted from: Cottrell, S., (2008) *The Study Skills Handbook*, Hampshire: Palgrave Macmillan, p286).

Descriptive writing	Critical analytical writing
States what happened	Identifies the significance of something that happened
States what something is like	Evaluates its strengths and weaknesses
States the order in which things happened	Structures information in order of importance
Explains what a theory says	Discusses the importance/failings/relevance of a theory in relation to a topic/idea
Explains how something works	Indicates why something will work (best)
Notes the methods used	Evaluates whether the extent to which the methods used were fit for purpose
Says when something occurred	Identifies why timing is of importance
States the different components	Weighs up the importance of component parts
States links between items	Shows the relevance of links between pieces of information
Gives information	Draws conclusions

11. A commentary on critiquing mathematics and statistics itself (optional)

11.1. Critiquing the technicality of statistics

In order to critique statistics itself we need to know statistics. The specific focus of the critique would then be on the method for collecting data, the appropriateness of the type of test used to analyse the data, the interpretations of the results (i.e. level of significance or the interval of confidence, or other), etc.

Since statistics is not my main area of maths I won't show an actual stats example for which I could produce a technical summary and critique. But I know enough to know which technical areas of statistics could be critiqued. Some of these are:

1. *Data collection*: What type of data are we collecting? Is the data of the right type for what we want to study, analyse and come to conclusions about? Have we collected a sufficient amount of data?
2. *Sampling techniques*: What type of sample are we going to choose? Random? Stratified? Some other type? Is our sampling procedure appropriate for what we are going to study, analyse and come to conclusions about?
3. *Probability distributions*: Are we studying a discrete distribution or a continuous one?
 - a) *Discrete distributions*: Examples include the Binomial, Geometric or Poisson distributions. Here one would need to consider how appropriate it was to use one distribution over another.

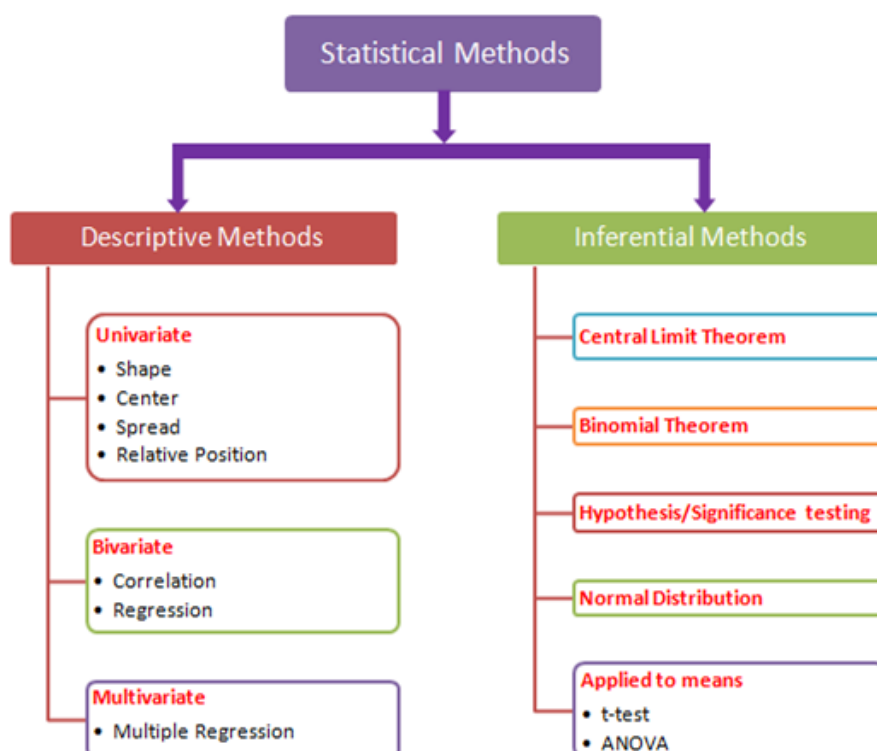
b) *Continuous distributions*: Examples include the normal distribution, the chi-squared distribution, the gamma distribution. As for the discrete case, one would need to consider how appropriate it was to use one distribution over another.

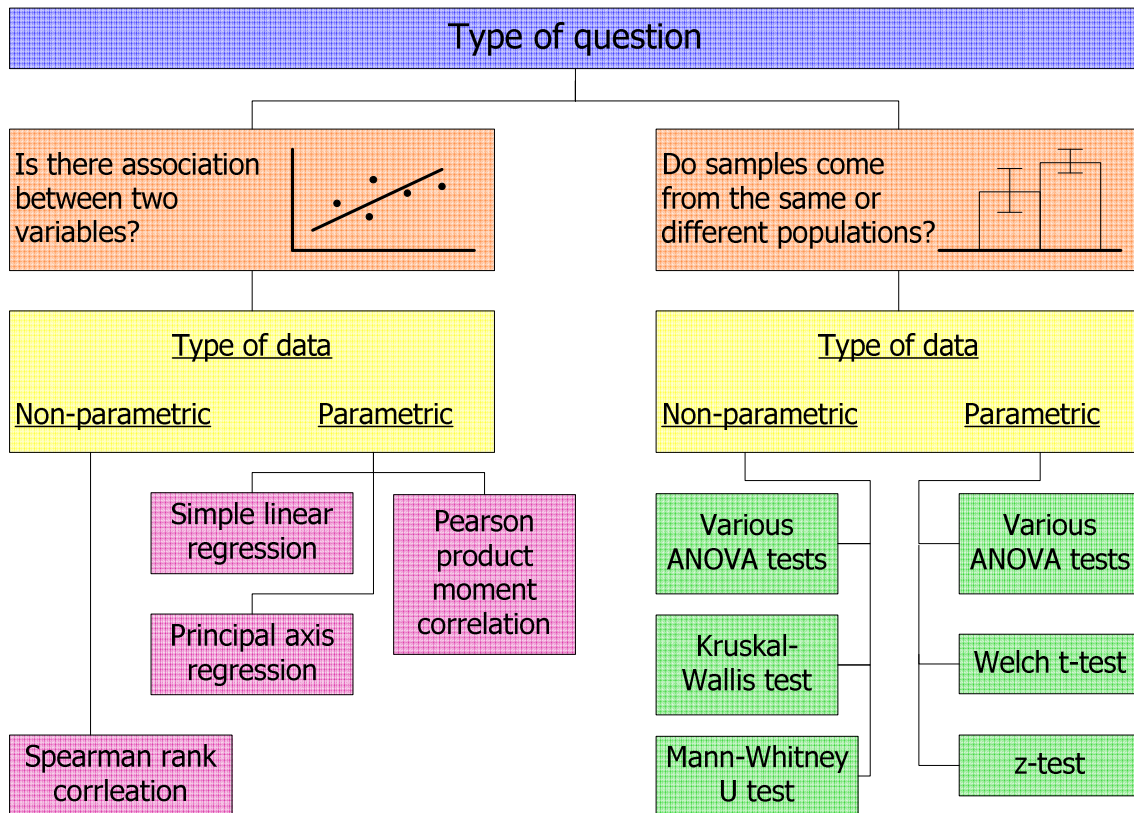
4. *Test statistic*: What statistic of the sample are we testing? The mean? The median? The standard deviation? Was the relevant spread considered? In other words, should we have used variance instead of standard deviation?
5. *Statistical analysis*: Was the appropriate analysis conducted? In other words, was testing statistical significance appropriate? Or would it have been more appropriate to set up a confidence interval?

If we are testing variance was the appropriate analysis of variance method used (there are several different types of methods for analysing variances).

6. *After all is said and done*: Have the results been clearly explained and appropriately interpreted? What is the significance of the results? What assumptions have been made? standard deviation? Was the relevant spread considered? In other words, should we have used variance instead of standard deviation.

Other technical aspects of statistics for which a critique could be provided is illustrated in the diagrams below. One point of contention when using statistics is usually whether or not the method used was appropriate for the type, and quantity, of data collected.





Another point of contention comes from the use of something called p -values which is the probability we need for some effect to be statistically significant and beyond mere chance. In fact, there is a great deal of contention in the academic community about the validity or relevance of using p -values (probability values which act as boundaries between significant and non-significant results), and associated significance levels. Such contention comes about as a result of, amongst other examples, the many medical trials conducted to prove the efficacy of a new drug, results which other people or organisations are unable to repeat or replicate. Similar problems of repeatability and replication of statistical results using p -values are seen in the social sciences. The reason for the contention is because *we* are the ones who choose the p -value for our experiments. It is *we* who decide where the boundary lies between results which will be considered statistically significant or not. P -values tend to be set as 0.1, 0.05 or 0.01 (i.e. a 10%, 5% or 1% level of significance), and a result which is statistically significant when we set one p -value may not be if we set another p -value.

State your hypothesis

Null hypothesis: A pizza company thinks customers eat an average of 4 slices per day.

Alternative hypothesis: You think the average is higher

Collect data

For this example, data collected on number of slices eaten gives an average of 5.6 slices

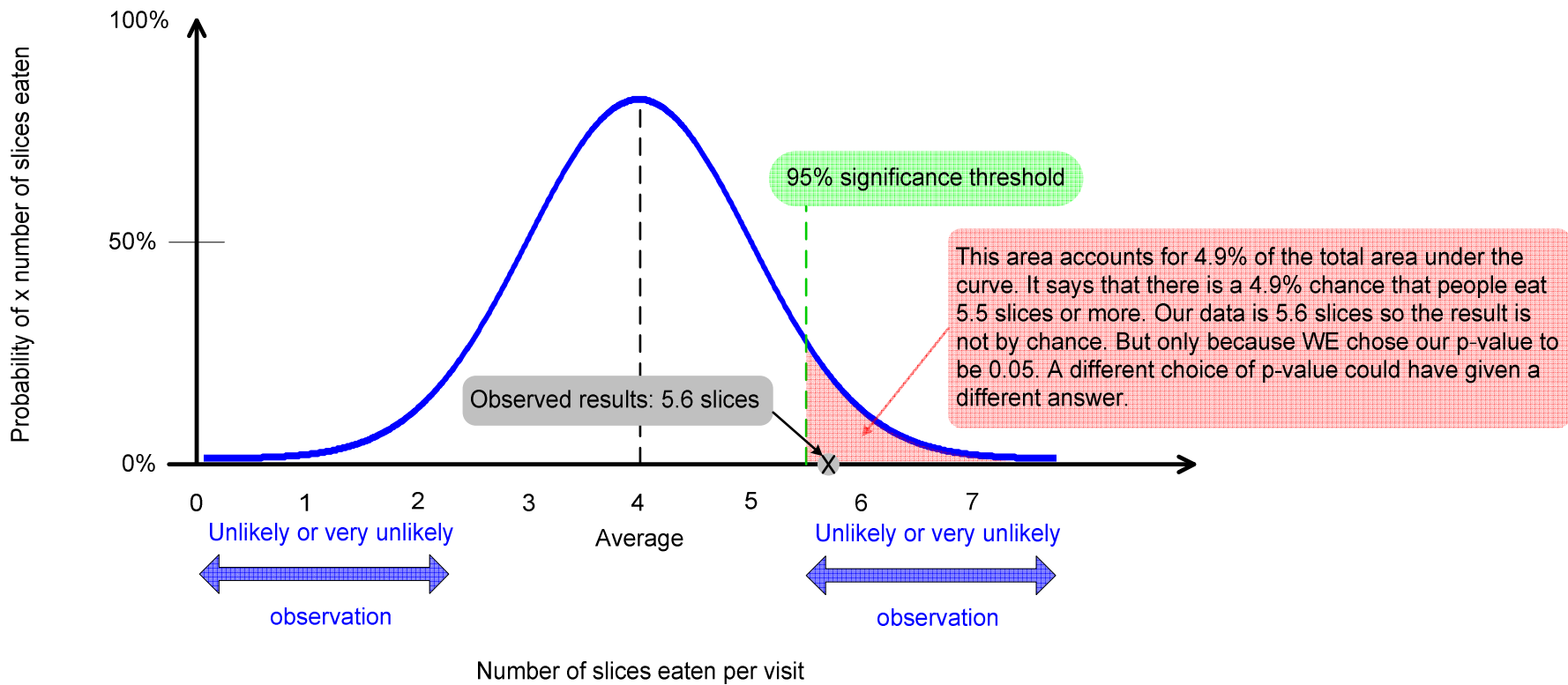
Test the result

Choose your significance level: $p\text{-value} = 0.05$ (5%)

Choose a test: for example z-test, t-test, or other

Is the result significant?

- $p > 0.1$: not significant
- $p < 0.1$: marginally significant
- $p < 0.05$: significant
- $p < 0.01$: very significant



11.2. Critiquing the technicalities of mathematics

When it comes to critiquing the technical aspects of a paper you need to know the topic of the paper. Again, since mathematics is the only topic I know well enough there is no getting around the fact that I will have to present a piece of maths in order to illustrate the critique of the technicalities of the mathematics itself. The first example is an example based on simple arithmetic so you should all be able to follow the maths of these.

Example 1: Various methods of multiplying two number

There are many ways of multiplying two numbers, some of which are shown below. Each has similarities and differences, its advantages and disadvantages. The different approaches to multiplication then allows us to develop a critique. We shall do this at the end of method 5.

Method 1: The usual way: Long multiplication

$$\begin{array}{r} 325 \\ \times \quad 12 \\ \hline 650 \\ 3250 \\ \hline 3900 \end{array}$$

Method 2: Repeated addition

$$325 + 325 + 325 + 325 + 325 + 325 + 325 + 325 + 325 + 325 + 325 + 325 = 3900$$

Method 3: Split the multiplier into a binomial term and expand

Version a

$$\begin{aligned} 325 \times 12 &= 325 \times (10 + 2) \\ &= 325 \times 10 + 325 \times 2 \\ &= 3250 + 650 \\ &= 3900 \end{aligned}$$

Version b

$$\begin{aligned} 325 \times 12 &= 325 \times (8 + 4) \\ &= 325 \times 8 + 325 \times 4 \\ &= 2600 + 1300 \\ &= 3900 \end{aligned}$$

Method 4: Halving and doubling.

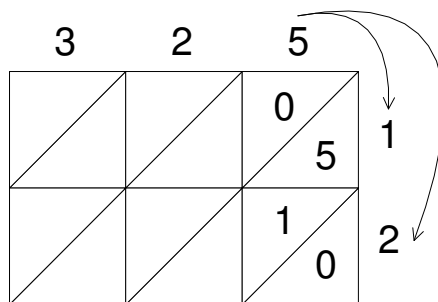
For 14×12 , halve one number (ignoring any remainder) until you get to 1, and double the other number. Then add all doubled numbers that lie across odd halved numbers:

14	12
7	24
3	48
1	96

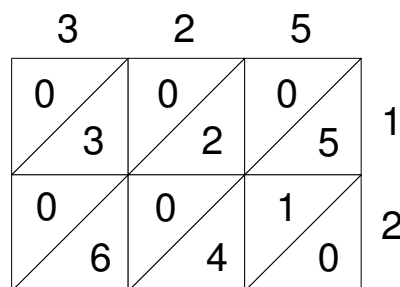
So $14 \times 12 = 24 + 48 + 96 = 168$

Method 5: Grid multiplication

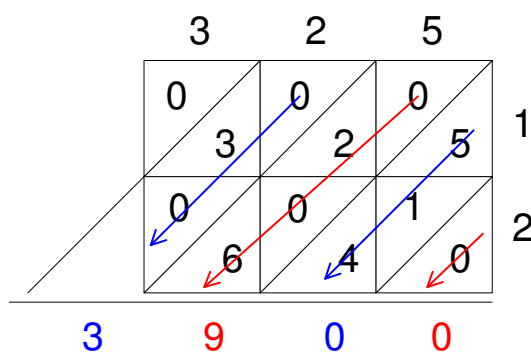
Multiply each digit with each other digit and place in the relevant triangle. For example



So we get



Finally, add along the diagonals, carrying as needed:



Now for the critique

What are the similarities and differences, advantages and disadvantages, etc. of these methods? Well ...

- 1) Method 1 = method 5 because the layout both numbers in hundreds, tens and units,
- 2) Method 1 \neq method 5 because (in one sense) method 5 allows use to multiply the digits independently of each other, whereas method does not (although this is not strictly true!)
- 3) Method 4 is completely different from all the other methods since it relies only on multiplying and dividing by 2, and taking account of the remainder of such a division.

etc.

Exercise: Below is another approach to multiplication. Compare and contrast this method with those above, and write a critique of this method.

$$97 \times 91 = 8827$$

$$\begin{array}{ccc} 100 - 97 & 100 - 91 & 100 - 12 = 88 \\ \downarrow & \downarrow & \uparrow \\ 3 & + & 9 & = & 12 \end{array}$$

and

$$3 \times 9 = 27$$

For your own interest, the following (which does not need to form part of your critique but can do so if you wish) justifies the mathematical validity of the above process:

$$\begin{aligned} 97 \times 91 &= (100 - 3)(100 - 9) \\ &= 100^2 - 100 \times 9 - 100 \times 3 + 3 \times 9 \\ &= 100^2 - 100(3 + 9) + 3 \times 9 \\ &= 100[100 - (3 + 9)] + 3 \times 9 \\ &= 100 \times 88 + 27 \\ &= 8827 \end{aligned}$$

Example 2: Solving a quadratic equation using the formula

We want to find the roots of $x^2 - 2x + 3 = 0$ using the quadratic formula. As such we can write

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(3)}}{2}.$$

Hence $x = -3$ or $x = +1$ are the roots of the quadratic. I can then critique the use of the formula, as a way of finding roots, as follows:

A critique of the use of the quadratic formula

The quadratic formula is ubiquitous when it comes to solving quadratic equations. Its great strength is that it will always work to find roots, real or complex. However, one could say that it is somewhat laborious. A possibly more efficient way in this particular example would have been to factorise by sight. This requires understanding the simple arithmetic combinations which can form the coefficients of the quadratic. This can be surprisingly simple to do once we have the knack of it. We can also note that, although factorisation always works in theory, in practice we usually want the coefficients of the quadratic to be simple enough for us to perform the mental arithmetic required. As an alternative to these two approaches, we could also *complete the square*. This procedure might be considered to be more complicated to use, but again it always provides an answer to the roots.

Exercise: Explain why the above is a critique, and identify all the linguistic structures used in this critique.

Example 3: On mathematics: Finding the max and min of a function using calculus

Find and classify all the turning points of the function $f(x) = (x - 1)(x + 1)(x + 2)$.

Solution

$$\begin{aligned} f(x) &= (x-1)(x+1)(x+2) \\ &= x^3 + 2x^2 - x - 2 \end{aligned}$$

$$\frac{dy}{dx} = 3x^2 + 4x - 1 = 0$$

$$\text{So } x = \frac{-4 \pm \sqrt{28}}{6} = -1.5485, 0.2153$$

<u>test</u> :	x :	-2	-1.5485	0	0.2153	1
	y' :	\	—	/	—	\

$$\therefore \text{max at } (0.2153, -2.1126)$$

$$\text{min at } (-1.5485, 0.6311)$$

Summary and critique

Identify in the commentary below all places where I am summarising and where I am critiquing.

In this solution we start by expanding the factored form of $f(x)$. The next step then finds the first derivative of this function. However, there is an inconsistent use of notation since the symbol 'y' has been used instead of $f(x)$. In terms of mathematical presentation it is important to be consistent in one's use of notation. Therefore, it would be more appropriate to continue with the same notation for the function as has been originally stated in the question, and write df/dx .

Also, the derivative is equated to zero at the same step as the derivative has been found. In general, when solving optimisation problems, these two aspects are presented as two different steps. In other words, it is more appropriate to first find the derivative to be

$$\frac{df}{dx} = 3x^2 + 4x - 1$$

and then, since we wish to find stationary points, we equate this derivative to zero:

$$3x^2 + 4x - 1 = 0$$

The next step involves simply solving this quadratic, from which we get the x ordinate of the stationary points.

After this comes the usual test for classifying stationary points. In this case the first derivative test has been used, but equally the second derivative test could have been used. Both tests have their advantages and disadvantages, namely that the second derivative test can sometimes be quicker, but may fail to give a valid answer in certain cases. The first derivative test will always work, but can take longer to apply, particularly if there are many turning points to have to test.

The last two steps of the solution classify the stationary points as well as giving their coordinates.

There is, however, a missing part to the solution. The question asked for all turning points of the function, and this solution has only presented three of the turning points (specifically the three that are called stationary points). It does not present the solution to the other two turning points which exist on this function, namely the point of inflection which exists between the maximum point and the minimum point. This is a serious omission in the presentation of a mathematical solution, and would need to be corrected before it could be considered a complete solution to the problem.

12. Critiquing the technicality of disciplines you know nothing about (optional)

None of us can present detailed critiques on any physical science discipline that are not our area of specialisation. However, we can still take a first pass at listing some of the technical aspects for critique that are specific to these disciplines? Beyond this it may be possible to foresee what might be critique-able in text, but this does depend on how technical a text is. In the next subsection present an example of critiquing a text the energy efficiency in building.

12.1. In-class exercise (if time allows)

Let us list some of the technical aspects which we could critique that are specific to the following disciplines. Choose the discipline relevant to you. If the discipline is not shown, then add it.

Mechanical engineering

1. = = =
2. = = =

Software engineering

1. = = =
2. = = =

Big data / Data analytics / Information security

1. = = =
2. = = =

Computer science

1. = = =
2. = = =

12.2. An example of critiquing a subject you know nothing about

The previous section brings us onto the issue of the degree to which it is possible to critique a subject we know nothing about. I contend that it is possible, to some degree, but obviously not to the depth of technicality that we could if we knew the topic.

To see how this might be, consider the following abstract taken from *An approach for the evaluation of energy and cost efficiency of glass facades*, Ikbal Cetiner, Ertan Ozkan, *Energy and Buildings* 37 (2005) 673–684.

Abstract

Glass facades, particularly in high-rise buildings, increase in energy consumption for heating, cooling and ventilation. This causes too high running cost of mechanical systems. Double skin glass facade is a system that decreases these disadvantages, by providing natural ventilation, preventing solar heat gain, controlling daylight, etc. This paper aims to investigate the appropriateness of double skin glass facades in moderate climate, such as Istanbul, in terms of the energy and cost efficiency when compared to single skin glass facades. For this purpose, an approach is proposed to determine the efficient alternatives. It comprises to generate standard facade alternatives by considering the objectives, constraints and performance criteria, and to evaluate their energy and cost efficiency for both single and double skin glass facades. In conclusion, the most energy efficient double skin glass facade is about 22.84% more efficient than the most energy efficient single skin glass facade is. Additionally, the most cost-efficient single skin glass facade is about 24.68% more efficient than the most cost efficient double skin glass facade is.

I can't provide a critique for this subject since I know nothing about the use of glass in buildings, the energy efficiency of glass, or life cycle costs in this context. But I can make an educated guess as to what I would consider critiquing, as follows:

- “Glass facades, particularly in high-rise buildings, increase in energy consumption for heating, cooling and ventilation. This causes too high running cost of mechanical systems. Double skin glass facade is a system that decreases these disadvantages, ...”

Here I could question the expense of creating double skin facades over single skin facades. It seems logical the double skin facades would be more expensive to

manufacture, but is this cost less than the total running costs in buildings having single skin facades?

- “Double skin glass facade is a system that decreases these disadvantages, by providing natural ventilation, preventing solar heat gain, controlling daylight, etc”

Here I could analyse the way in which they measured natural ventilation, reduction in solar heat gain, and the control of daylight in order to find any better ways in which this might be done.

- “This paper aims to investigate the appropriateness of ...” How do they measure or investigate “appropriateness”? Is their investigation appropriate? This is something I could look at.
- “For this purpose, an approach is proposed to determine the efficient alternatives. It comprises to generate standard facade alternatives by considering the objectives, constraints and performance criteria ...”

Are objectives, constraints and performance criteria the relevant aspects to consider in the context of single/double glass facades? If not why not? And how could things be done better?

The following is where most of your critiquing would be directed:

- “... the most energy efficient double skin glass facade is about 22.84% more efficient than the most energy efficient single skin glass facade is. Additionally, the most cost efficient single skin glass facade is about 24.68% more efficient than the most cost efficient double skin glass facade is.”

The authors states percentage gains/improvements and energy and cost efficiencies. Here you would have to analyse and understand how they came to these numbers.

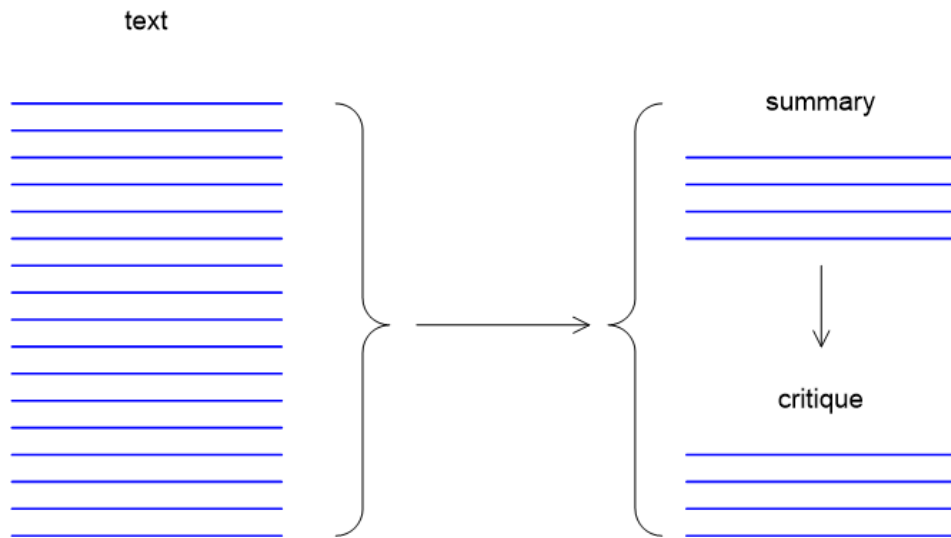
In other words, you would need to repeat their analysis according to your expertise to see if they have made any mistakes, assumptions, simplifications, omissions, (if any) etc. in their analysis, and then make recommendations as to how they could improve their work.

Exercise: Using any paper(s) of your choice identify the aspects for critique that are specific to your discipline. Compare these with your classmate’s version. Are there any similarities and differences between your list and their list?

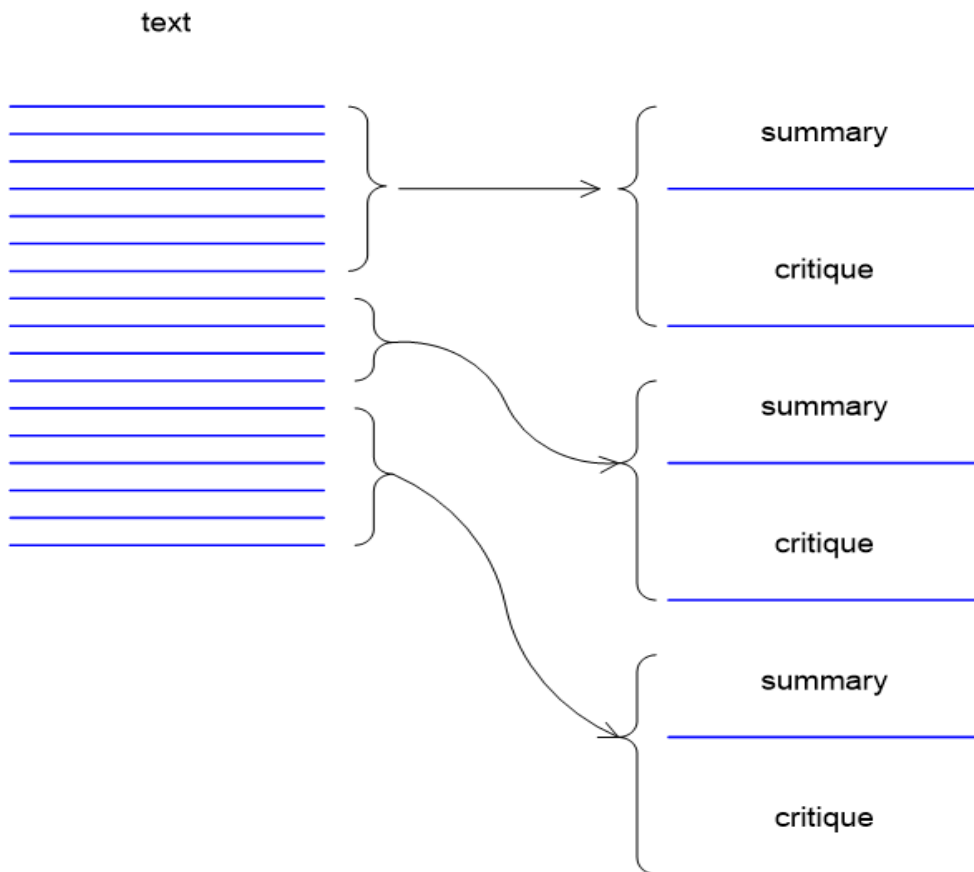
13. Possible ways of organising summaries and critiques

Below are diagrams which I have invented in order to illustrate possible ways of structuring one's summaries and critiques in an essay.

1) Summarise the text then critique the text:



2) Summarise and critique one part of the text at a time:



14. Appendix: Suggested answers to the exercise on p39

Mechanical engineering

1. Fluid mechanics;
2. Properties of materials;
3. Vibration analysis;

Software engineering

1. Design of software: UML, SSADM, other methodology;
2. User requirements: Who is to use the software, how do they want to use it ...;
3. Languages: Pascal, Java, .Net ... Python (to do with efficiency consideration?) ;
4. Testing: Beta testing and other forms of testing;

Information security

1. Phishing, hacking, identity theft, etc.;
2. Information security awareness programs.;

Computer science

1. Hardware architecture;
2. Formal methods;

Mechatronics

1. Design methods?
2. Requirements: Why prioritise one requirement over another requirement?
3. Materials necessary to build: why this motor instead of another?
4. Testing: unit testing vs whole testing?
5. Which coding language to use? C or Python?
6. How to interface software with hardware?